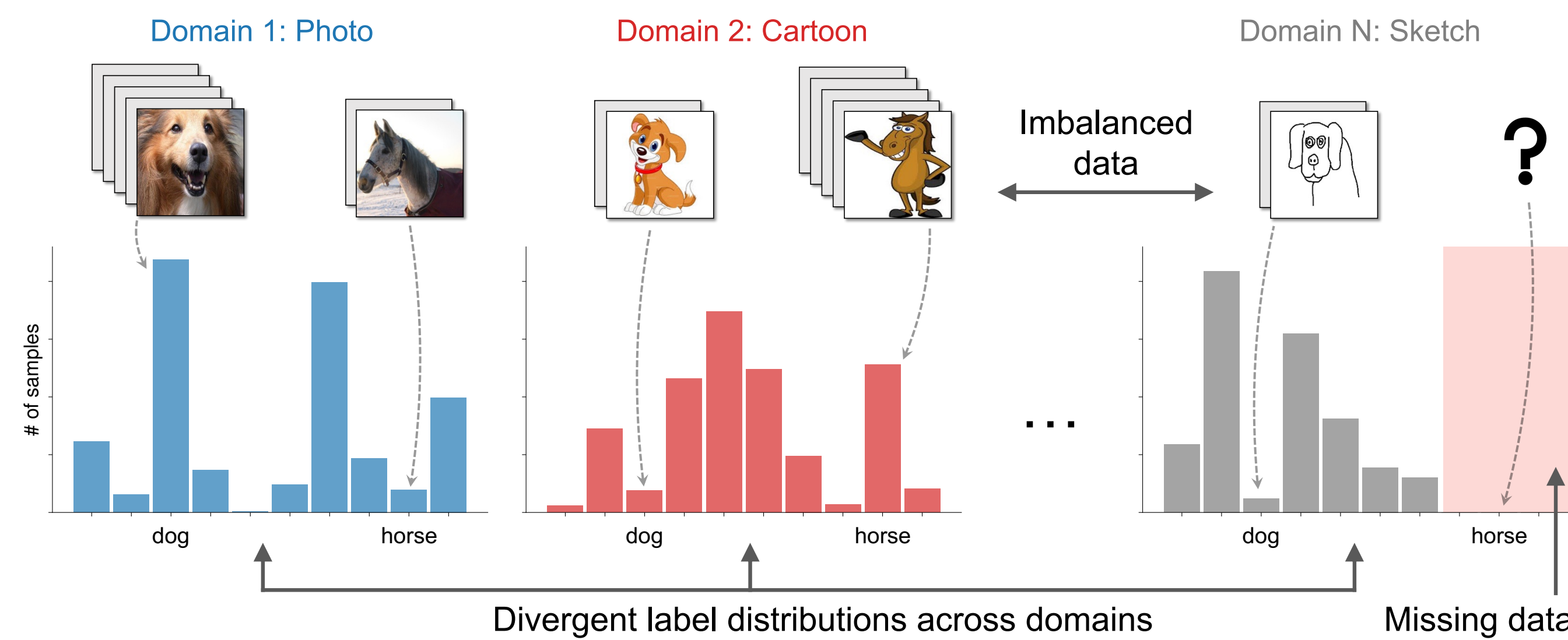


## Background & Motivation

- Existing studies on data imbalance focus on *single domain*
- Yet, data for one task can originate from distinct *domains*
- A minority class in one domain can have many samples from other domains, which can help for generalization

## Multi-Domain Long-Tailed Recognition (MDLT)

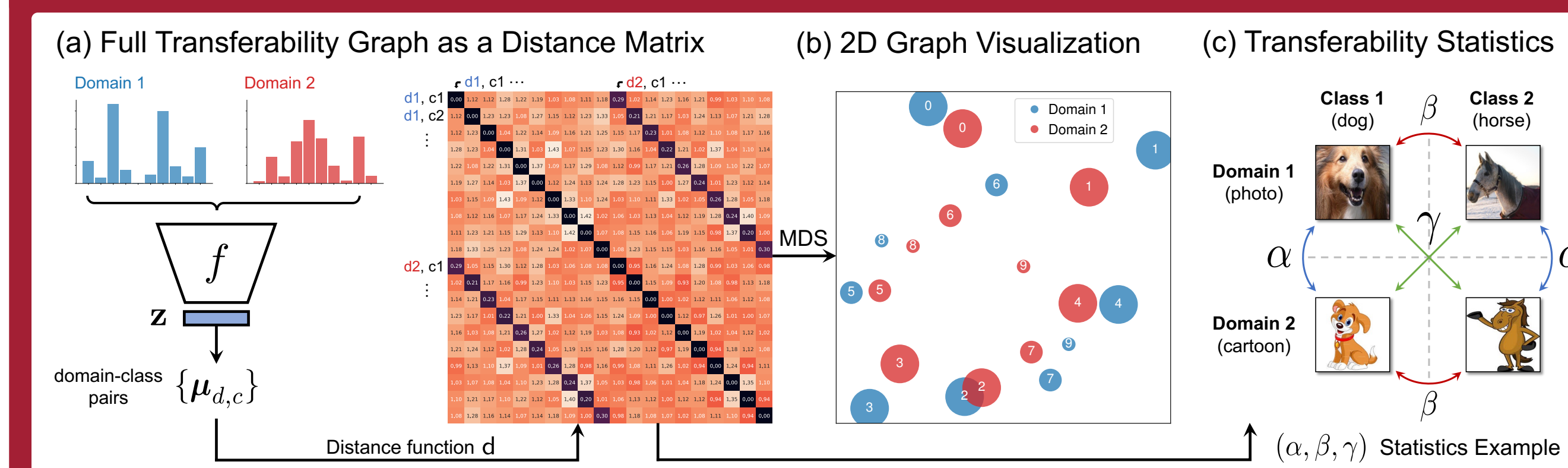


MDLT learns from multi-domain imbalanced data, tackles *label imbalance*, *domain shift*, and *divergent label distributions across domains*, and generalize to *all* domain-class pairs

### Challenges:

- Different label distributions for each of the domains
- Multi-domain data inherently involves *domain shift*
- Zero-shot generalization *within* and *across* domains

## Domain-Class Transferability Graph

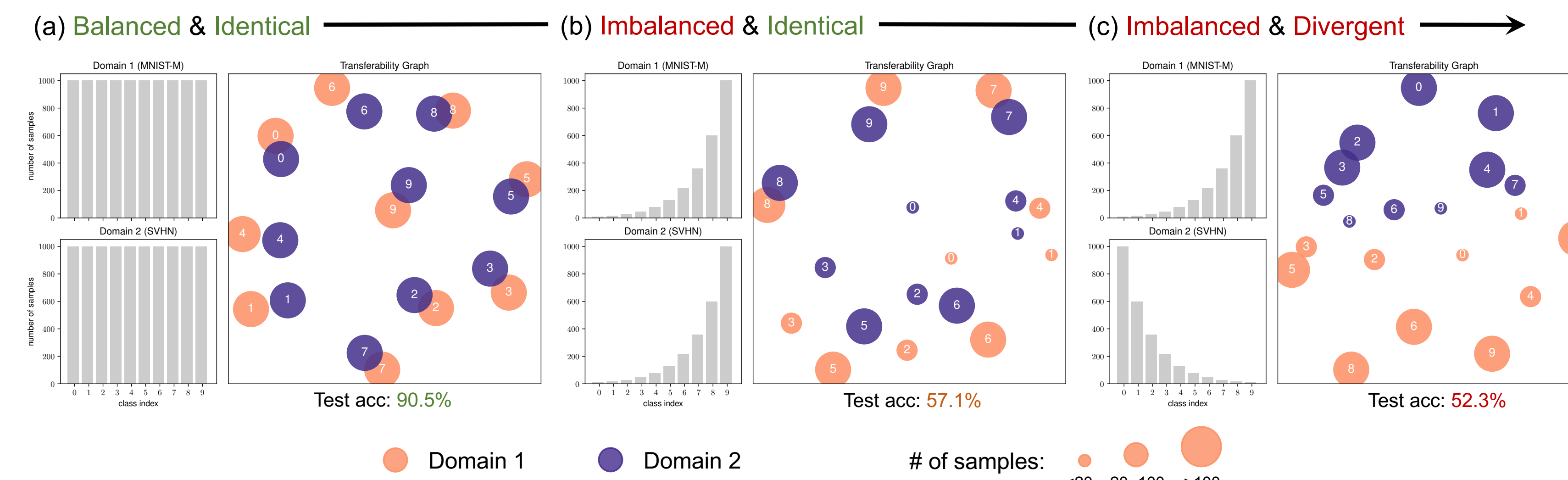


**Definition 1: Transferability from  $(d, c)$  to  $(d', c')$**   
 $\text{trans}((d, c), (d', c')) \triangleq \mathbb{E}_{\mathbf{z} \in \mathcal{Z}_{d,c}} [d(\mathbf{z}, \boldsymbol{\mu}_{d',c'})]$ ,  $\boldsymbol{\mu}_{d',c'} \triangleq \mathbb{E}_{\mathbf{z}' \in \mathcal{Z}_{d',c'}} [\mathbf{z}']$ .

**Definition 2:  $(\alpha, \beta, \gamma)$  Transferability Statistics**

- Diff domains, same class:  $\alpha = \mathbb{E}_c \mathbb{E}_d \mathbb{E}_{d' \neq d} [\text{trans}((d, c), (d', c))]$ .
- Same domain, diff classes:  $\beta = \mathbb{E}_d \mathbb{E}_c \mathbb{E}_{c' \neq c} [\text{trans}((d, c), (d, c'))]$ .
- Diff domains, diff classes:  $\gamma = \mathbb{E}_d \mathbb{E}_{d' \neq d} \mathbb{E}_c \mathbb{E}_{c' \neq c} [\text{trans}((d, c), (d', c'))]$ .

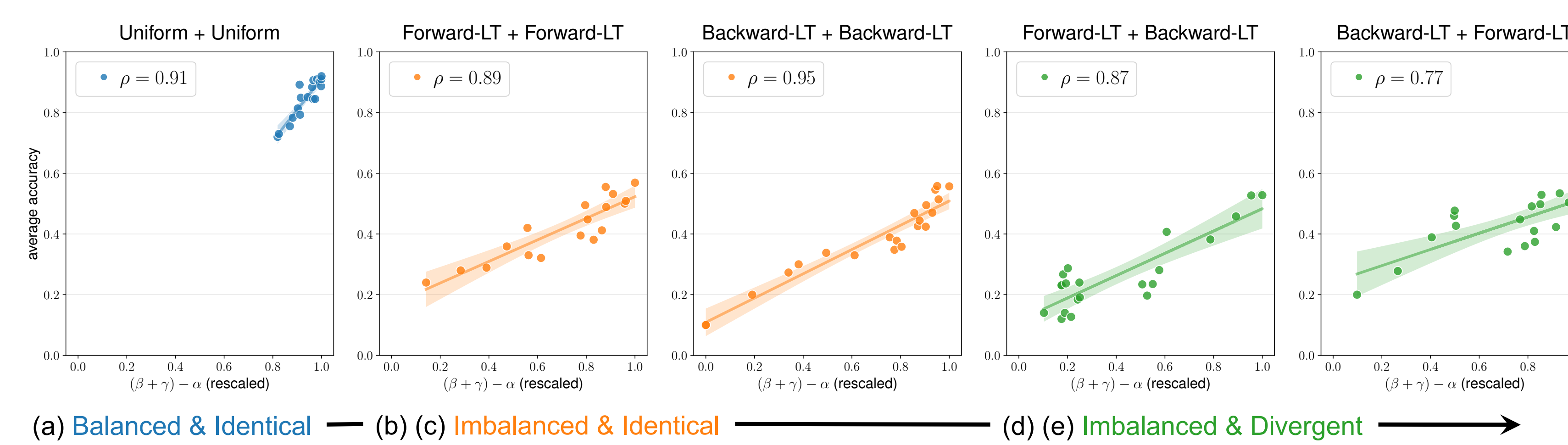
## Observation #1: Divergent Label Distributions Hamper Transferable Features



- Digits-MLT, a two-domain toy MDLT dataset
- ResNet-18 trained using ERM with different label distributions across domains (gray histograms)
- Different patterns of learned transferability graph:
  - Balanced & Identical*: transferable features, high accuracy
  - Imbalanced & Identical*: transferable features (majority better than minority classes), moderate accuracy
  - Imbalanced & Divergent*: features no longer transferable; clear gap across domains; worst accuracy

**Implication:** Transferable features needed in MDLT.

## Observation #2: Transferability Statistics Characterize Generalization



- Different label configurations for Digits-MLT:
  - Uniform / Forward-LT / Backward-LT
- 20 ERM models with varying hyperparameters trained for each configuration (each dot a model)
- Plot test accuracy against  $(\beta + \gamma - \alpha)$  quantity:
  - Strong correlation across all ranges / label configurations
  - Imbalance boosts risk of learning less transferable features

**Implication:**  $(\alpha, \beta, \gamma)$  statistics characterize model performance in MDLT.

## BoDA: A Loss that Bounds the Transferability Statistics

**Recall:**  $(\alpha, \beta, \gamma)$  statistics governs the success in MDLT – smaller  $\alpha$  and larger  $\beta, \gamma$  lead to better model performance

**A First Approach: Domain-Class Distribution Alignment ( $\mathcal{L}_{DA}$ )**

$$\mathcal{L}_{DA}(\mathcal{Z}, \{\boldsymbol{\mu}\}) = \sum_{\mathbf{z}_i \in \mathcal{Z}} \frac{-1}{|\mathcal{D}| - 1} \sum_{d \in \mathcal{D} \setminus \{d_i\}} \log \frac{\exp(-d(\mathbf{z}_i, \boldsymbol{\mu}_{d,c_i}))}{\sum_{(d',c') \in \mathcal{M} \setminus \{(d_i,c_i)\}} \exp(-d(\mathbf{z}_i, \boldsymbol{\mu}_{d',c'}))}$$

- Pros:** tackles label *divergence* – numerator  $\rightarrow$  *positive* cross-domain pairs ( $\alpha$ ); denominator  $\rightarrow$  *negative* cross-class pairs ( $\beta, \gamma$ )
- Cons:** does not address label *imbalance* – independent of the number of samples in each  $(d, c)$ , thus dominated by majority  $(d, c)$

**Balanced Domain-Class Distribution Alignment (BoDA).**

$$\mathcal{L}_{BoDA}(\mathcal{Z}, \{\boldsymbol{\mu}\}) = \sum_{\mathbf{z}_i \in \mathcal{Z}} \frac{-1}{|\mathcal{D}| - 1} \sum_{d \in \mathcal{D} \setminus \{d_i\}} \log \frac{\exp(-\bar{d}(\mathbf{z}_i, \boldsymbol{\mu}_{d,c_i}))}{\sum_{(d',c') \in \mathcal{M} \setminus \{(d_i,c_i)\}} \exp(-\bar{d}(\mathbf{z}_i, \boldsymbol{\mu}_{d',c'}))}, \bar{d}(\mathbf{z}_i, \boldsymbol{\mu}_{d,c}) = \frac{d(\mathbf{z}_i, \boldsymbol{\mu}_{d,c})}{N_{d,c_i}}$$

- BoDA scales original  $d$  by a factor of  $1/N_{d,c_i}$  – it counters the effect of imbalanced  $(d, c)$  by introducing a *balanced* distance  $\bar{d}$ .

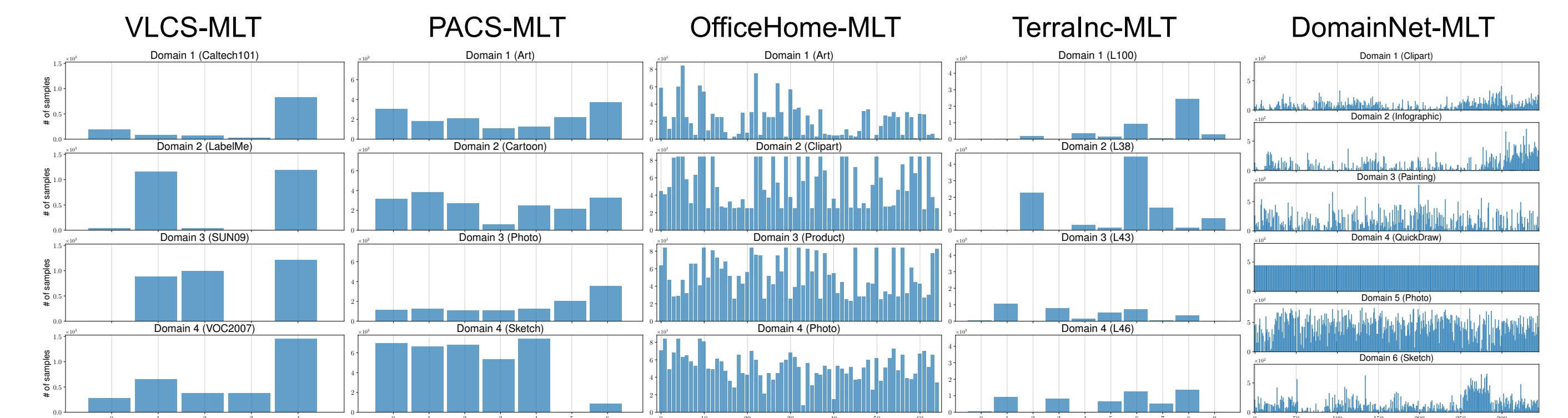
**Theorem 1 ( $\mathcal{L}_{BoDA}$  as an Upper Bound).** Given a multi-domain long-tailed dataset  $\mathcal{S}$  with domain label space  $\mathcal{D}$  and class label space  $\mathcal{C}$  satisfying  $|\mathcal{D}| > 1$  and  $|\mathcal{C}| > 1$ , let  $(\alpha, \beta, \gamma)$  be the transferability statistics for  $\mathcal{S}$ . It holds that

$$\mathcal{L}_{BoDA}(\mathcal{Z}, \{\boldsymbol{\mu}\}) \geq N \log \left[ |\mathcal{D}| - 1 + |\mathcal{D}|(|\mathcal{C}| - 1) \exp \left( \frac{|\mathcal{C}||\mathcal{D}|}{N} \cdot \alpha - \frac{|\mathcal{C}|}{N} \cdot \beta - \frac{|\mathcal{C}|(|\mathcal{D}| - 1)}{N} \cdot \gamma \right) \right].$$

**Implication #1:**  $\mathcal{L}_{BoDA}$  upper-bounds  $(\alpha, \beta, \gamma)$  statistics in a desired form that naturally translates to better performance.

**Implication #2:** The constant factors correspond to how much each component contributes to the transferability graph.

## MDLT Benchmarks + Results



5 MDLT benchmark datasets / ~20 baseline algorithms

Algorithm	VLCS-MLT	PACS-MLT	OfficeHome-MLT	TerraInc-MLT	DomainNet-MLT	Avg
ERM	76.3 ±0.4	97.1 ±0.1	80.7 ±0.0	75.3 ±0.3	58.6 ±0.2	77.6
Current SOTA	75.9 ±0.5	96.6 ±0.5	81.9 ±0.1	76.4 ±0.5	59.4 ±0.1	78.0
BoDA	78.2 ±0.4	97.1 ±0.2	82.4 ±0.2	83.0 ±0.4	61.7 ±0.2	80.5
BoDA vs. ERM	+1.9	+0.1	+1.7	+7.7	+3.1	+2.9

## Beyond MDLT: Domain Generalization

- Domain generalization (DG)**
  - Learn from multiple domains & generalize to *unseen* domains
- Data imbalance is an *intrinsic* problem in DG**
  - Learning domains naturally differ in their label distributions
  - Domains can have (severe) class imbalance within each domain

BoDA establishes new SOTA on DG benchmarks

Algorithm	VLCS	PACS	OfficeHome	TerraInc	DomainNet	Avg
ERM	77.5 ±0.4	85.5 ±0.2	66.5 ±0.3	46.1 ±1.8	40.9 ±0.1	63.3
Current SOTA	78.8 ±0.6	86.2 ±0.3	68.7 ±0.3	47.6 ±1.0	41.5 ±0.1	64.5
BoDA	78.5 ±0.3	86.9 ±0.4	69.3 ±0.1	50.2 ±0.4	42.7 ±0.1	65.5
BoDA + Current SOTA	79.1 ±0.1	87.9 ±0.5	69.9 ±0.2	50.7 ±0.6	43.5 ±0.3	66.2
BoDA vs. ERM	+1.6	+2.4	+3.4	+4.6	+2.6	+2.9

**Implication:** Label imbalance affects out-of-distribution generalization, and is crucial for DG algorithm design.

## Conclusion & More Information

New Task	New Techniques	New Benchmarks
MDLT	Domain-Class Trans. Graph BoDA	5 MDLT datasets

**Code:** <https://github.com/YyzHarry/multi-domain-imbalance>

**Project page:** <http://mdlt.csail.mit.edu/>